6 ALGEBRA PLAY

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6.1 Algebra Play

Over the last two years, we have used algebra to model different situations. We have learned how to solve algebraic equations and find the values of unknown letter-numbers. Let's now have some fun with algebra. We shall investigate tricks and puzzles, and explain why they work using algebra. We will also see how to invent new tricks and puzzles to entertain others.

6.2 Thinking about 'Think of a Number' Tricks

In Grade 7, we learned about 'Think of a Number' tricks, like this one.

1. Think of a number.
2. Double it.
3. Add four.
4. Divide by two.
5. Subtract the original number you thought of.

? I predict you get 2. Am I right? Try it out with different starting numbers. Do you always end up with the same value, 2? Why?

We saw that we can understand such tricks through algebra.

1. Think of a number: x
2. Double it: $2x$
3. Add four: $2x + 4$
4. Divide by 2: $x + 2$
5. Subtract the original number you thought of: $x + 2 - x = 2$

Therefore, no matter what the starting number is, the end result will always be 2!

? How would you change this game to make the final answer 3? What about 5?

? Can you come up with more complicated steps that always lead to the same final value?

Let us now look at a different trick of this type.

? How did Shubham figure out the date chosen by Mukta?

- Let the month be M and the day be D.
- Multiply M by 5: $5M$

- Add 6: $5M + 6$
- Multiply by 4: $20M + 24$
- Add 9: $20M + 33$
- Multiply by 5: $100M + 165$
- Add the day: $100M + 165 + D$

Mukta's answer was 291.

- $291 = 100M + 165 + D$
- $291 – 165 = 100M + D$
- $126 = 100M + D$

Since D is a day within a month, it is atmost 31 and requires only 2 digits. So the last 2 digits are D and what comes before that is M. In this case, M is 1 and D is 26, i.e., the 26th of January.

? Mukta thinks of another date, follows the same steps, and reports her answer as 1390. What date did Mukta start with this time?

- Subtracting 165 from 1390, we get 1225.
- This means that the date she thought of was 25th December.

? Find the dates if the final answers are the following:

(i) 1269

(ii) 394

(iii) 296

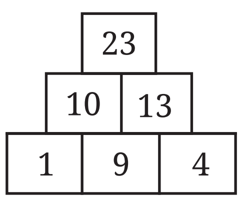
You can try this trick with your friends. Ask them to choose the starting date as their birthday.

? Can you change the steps in this trick and still find the original date? Instead of subtracting 165 from the final answer, you might have to subtract some other number.

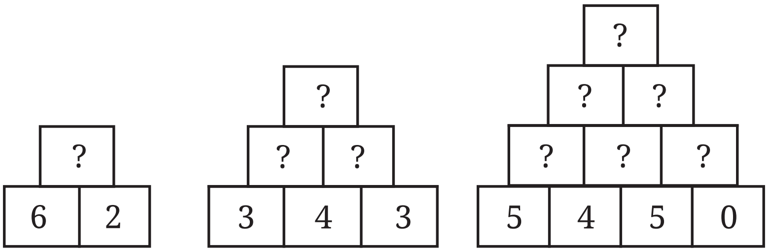
? Try to devise your own 'Think of a Number' trick.

6.3 Number Pyramids

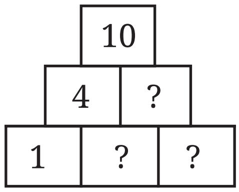
In a number pyramid, each number is the sum of the two numbers directly below it (see the figure).

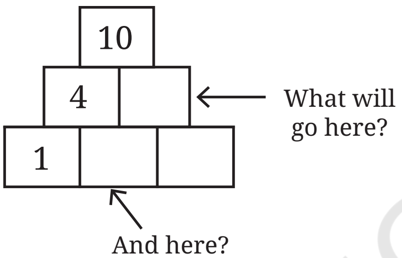


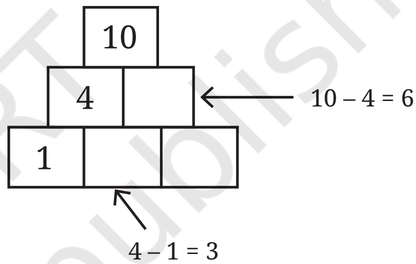
? Use the same rule to fill these pyramids:

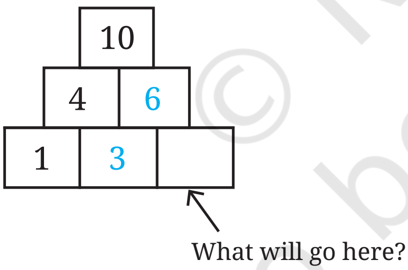


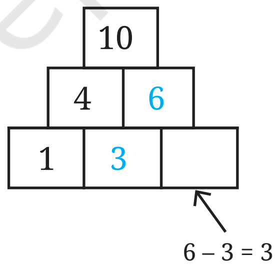
? How do we fill this pyramid?

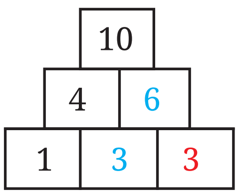




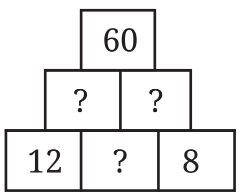




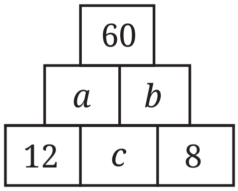




? What about filling in the numbers in this pyramid? Where do we start?



Let us fill the empty boxes with letter-numbers.



From the rules for filling up pyramids, we get the following equations.

- $a + b = 60$
- $12 + c = a$
- $c + 8 = b$

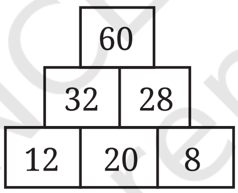
From this, we see that,

- $(12 + c) + (c + 8) = a + b = 60$

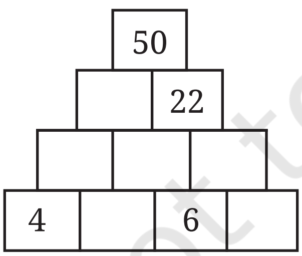
Hence,

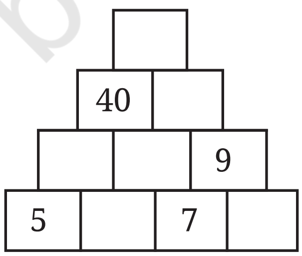
- $20 + 2c = 60$
- $2c = 60 - 20 = 40$
- $c = 20$.

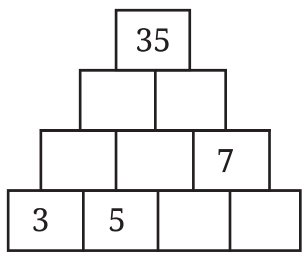
Once we know c, we can find a and b, and complete the pyramid.



? Fill the following pyramids:

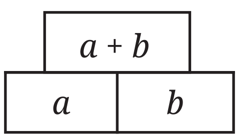






? What is the relationship between the numbers in the bottom row and the number at the top?

Let us start with the simplest pyramid.



What about a pyramid with three rows?

Using letter numbers for the bottom row, we can write an expression for the top row.

	$a + 2b + c$	
$a + b$		$b + c$
a	b	c

Figure it Out

1. Without building the entire pyramid, find the number in the topmost row given the bottom row in each of these cases.

4	13	8

7	11	3

10	14	25

2. Write an expression for the topmost row of a pyramid with 4 rows in terms of the values in the bottom row.

3. Without building the entire pyramid, find the number in the topmost row given the bottom row in each of these cases.

8	19	21	13

7	18	19	6

9	7	5	11

Recall the Virahāṅka-Fibonacci number sequence 1, 2, 3, 5, … where each number is the sum of the two numbers before it.

4. If the first three Virahāṅka-Fibonacci numbers are written in the bottom row of a number pyramid with three rows, fill in the rest of the pyramid. What numbers appear in the grid? What is the number at the top? Are they all Virahāṅka-Fibonacci numbers?

5. What can you say about the numbers in the pyramid and the number at the top in the following cases?

   (i) The first four Virahāṅka-Fibonacci numbers are written in the bottom row of a four row pyramid.

   (ii) The first 29 Virahāṅka-Fibonacci numbers are written in the bottom row of a 29 row pyramid.

6. If the bottom row of an n row pyramid contains the first n Virahāṅka-Fibonacci numbers, what can we say about the numbers in the pyramid? What can we say about the number at the top?

6.4 Fun with Grids

Calendar Magic

A page from a calendar is given below. Your friend picks a 2 × 2 grid from this calendar, adds the 4 numbers in this grid and tells you the sum.

AUGUST 2025						
SUN	MON	TUE	WED	THU	FRI	SAT
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

6	7
13	14

6 + 7 + 13 + 14 = 40

Can we find the 4 numbers in the grid from just knowing this sum?

Let us use algebra. Consider a 2 × 2 grid. Let a represent the top left number. What are the other numbers in terms of a?

a	$a + 1$
$a + 7$	$a + 8$

Adding all four numbers, the sum is $a + (a + 1) + (a + 7) + (a + 8) = 4a + 16$.

Suppose you are told that the sum is 36. Can you find the 4 numbers in the grid?

- $4a + 16 = 36$
- $4a = 20$ (subtracting 16 from both sides)
- $a = 5$ (dividing both sides by 4)

Now that we have found a, we know that the other three numbers are $a + 1$, $a + 7$ and $a + 8$. Therefore, the grid must be the following:

5	6
12	13

? Create your own calendar trick. For instance, choose a grid of a different size and shape.

Math Talk

AUGUST 2025						
SUN	MON	TUE	WED	THU	FRI	SAT
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

2	3	4	5	6	7	8	9	10
12	13	14	15	16	17	18	19	20
22	23	24	25	26	27	28	29	30
32	33	34	35	36	37	38	39	40
42	43	44	45	46	47	48	49	50

Algebra Grids

In the following grid, shapes represent numbers. In each row, the last column is the sum of the values to its left. How do we find the values of the shapes?

■	■	■	27
●	●	■	19

■ + ■ + ■ = 27

So, ■ = 9

Now, ● + ● + ■ = 19

● + ● + 9 = 19

2 × ● + 9 = 19

● = 5

? In the following grids, find the values of the shapes and fill in the empty squares:

■	■	●	27
●	●	■	21
●	■	●	

●	◆	◆	18
◆	●	●	15
◆	●	●	

6.5 The Largest Product

? Fill the digits 2, 3, and 5 in ☐☐ × ☐, using each digit once. What is the largest product possible?

Let us approach this problem systematically. There are six ways to place three digits:

- We can fill the first box with 2, 3 or 5.
- For each of these choices, we have 2 ways of filling the remaining 2 digits.
- The six options are 23 × 5, 25 × 3, 32 × 5, 35 × 2, 52 × 3, 53 × 2.

? How do we find the largest product among these six options?

We can group them in pairs where the multiplier is the same.

- 35 × 2 and 53 × 2
- 25 × 3 and 52 × 3
- 23 × 5 and 32 × 5

In each pair, the one with the larger multiplicand generates the larger product, so we can reduce the comparison to these three expressions.

- 53 × 2
- 52 × 3
- 32 × 5

It is clear that 52 × 3 is bigger than 53 × 2, so we only need to compare 52 × 3 and 32 × 5. Let us expand these.

- 32 × 5 = (3 × 10 × 5) + (2 × 5)
- 52 × 3 = (5 × 10 × 3) + (2 × 3)

The first terms in both expressions are the same. The second term shows that 32 × 5 is larger, and hence the largest of the six possible products we can form with 2, 3 and 5.

? In this case, we used the largest digit as the multiplier. The other two digits were arranged in decreasing order to form the multiplicand. Will this always be the case? Let us find out using algebra.

Suppose p, q, and r are the three digits such that $p < q < r$.

As before, we have six possible products, which we group by the multiplier:

- $qr \times p$, $rq \times p$
- $pr \times q$, $rp \times q$
- $pq \times r$, $qp \times r$

In each pair, the multiplicand with the larger tens digit forms the larger product. So we have three products to compare:

- $rq \times p$
- $rp \times q$
- $qp \times r$

Since $q > p$, we can see that $rp \times q$ is bigger than $rq \times p$. This leaves us with a comparison between $qp \times r$ and $rp \times q$. If we expand these, we get

- $qp \times r = (10 \times q \times r) + (p \times r)$
- $rp \times q = (10 \times r \times q) + (p \times q)$

Once again, the first term is the same in both expressions. Since $r > q$ the second term of the first expression is larger, so the largest product is $qp \times r$. This matches our earlier observation that the largest digit should be the multiplier and the other two digits should be arranged in decreasing order to form the multiplicand.

Figure it Out

1. Fill the digits 1, 3, and 7 in ☐☐ × ☐ to make the largest product possible.
2. Fill the digits 3, 5, and 9 in ☐☐ × ☐ to make the largest product possible.

6.6 Decoding Divisibility Tricks

It is now Mukta's turn to show a mathematics trick to Shubham.

If we choose other 2-digit numbers, and follow the steps, will there always be no remainder?

Let's try to understand how Mukta's trick works.

Suppose the two-digit number is *ab*. When it is reversed, the new number is *ba*.

- If $b > a$, then $ba > ab$. So, the difference is

  $(10b + a) - (10a + b)$

  $= 10b - b - 10a + a$

  $= 9b - 9a = 9\,(b - a)$.

The difference is divisible by 9.

Can you work out what happens if $a > b$?

Figure it Out

1. In the trick given above, what is the quotient when you divide by 9? Is there a relationship between the two numbers and the quotient?

2. In the trick given above, instead of finding the difference of the two 2-digit numbers, find their sum. What will happen? For example:
   - We start with 31. After reversing we get 13. Adding 31 and 13, we get 44.
   - We start with 28. After reversing we get 82. Adding 28 and 82, we get 110.
   - We start with 12. After reversing we get 21. Adding 12 and 21, we get 33.

   Observe that all these numbers are divisible by 11. Is this always true? Can we justify this claim using algebra?

3. Consider any 3-digit number, say *abc* ($100a + 10b + c$). Make two other 3-digit numbers from these digits by cycling these digits around, yielding *bca* and *cab*. Now add the three numbers. Using algebra, justify that the sum is always divisible by 37. Will it also always be divisible by 3? [**Hint:** Look at some multiples of 37.]

4. Consider any 3-digit number, say *abc*. Make it a 6-digit number by repeating the digits, that is *abcabc*. Divide this number by 7, then by 11, and finally by 13. What do you get? Try this with other numbers. Figure out why it works. [**Hint:** Multiply 7, 11 and 13.]

Math Talk

5. There are 3 shrines, each with a magical pond in the front. If anyone dips flowers into these magical ponds, the number of flowers doubles. A person has some flowers. He dips them all in the first pond and then places some flowers in shrine 1. Next, he dips the remaining flowers in the second pond and places

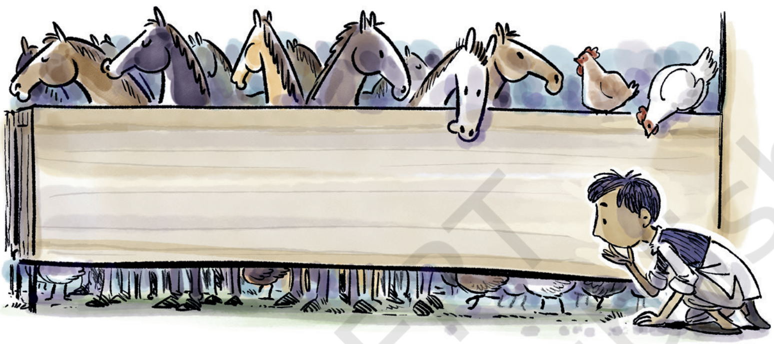
some flowers in shrine 2. Finally, he dips the remaining flowers in the third pond and then places them all in shrine 3. If he placed an equal number of flowers in each shrine, how many flowers did he start with? How many flowers did he place in each shrine?

6. A farm has some horses and hens. The total number of heads of these animals is 55 and the total number of legs is 150. How many horses and how many hens are on the farm?

   Can you solve this without letter-numbers?

   Math Talk

   [**Hint:** If all the 55 animals were hens, then how many legs would there be? Using the difference between this number and 150, can you find the number of horses?]



7. A mother is 5 times her daughter's age. In 6 years' time, the mother will be 3 times her daughter's age. How old is the daughter now?

8. Two friends, Gauri and Naina, are cowherds. One day, they pass each other on the road with their cows. Gauri says to Naina, "You have twice as many cows as I do". Naina says, "That's true, but if I gave you three of my cows, we would each have the same number of cows". How many cows do Gauri and Naina have?

9. I run a small dosa cart and my expenses are as follows:
   - Rent for the dosa cart is ₹5000 per day.
   - The cost of making one dosa (including all the ingredients and fuel) is ₹10.

   (i) If I can sell 100 dosas a day, what should be the selling price of my dosa to make a profit of ₹2000?

   (ii) If my customers are willing to pay only ₹50 for a dosa, how many dosas should I aim to sell in a day to make a profit of ₹2000?

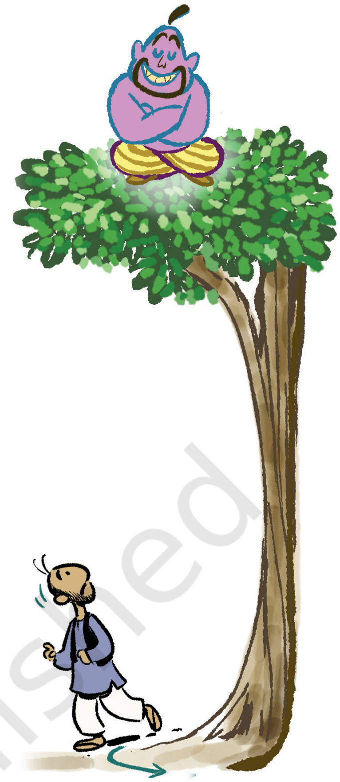
10. Evaluate the following sequence of fractions:

    $\frac{1}{3}, \frac{(1+3)}{(5+7)}, \frac{(1+3+5)}{(7+9+11)}$

    What do you observe? Can you explain why this happens?

    [**Hint:** Recall what you know about the sum of the first n odd numbers.]

11. **Karim and the Genie**



Karim was taking a nap under a tree. He had a dream about a magical lamp and a genie. He heard a voice saying, "I have come to serve you, Oh master". He woke up and to his surprise, it was a genie!

"Do you want to make money?", asked the genie. Karim nodded dumbly in bewilderment. The genie continued, "Do you see the banyan tree over there? All you have to do is go around it once. The money in your pocket will double".

Karim immediately started towards the tree, only to be stopped by the genie. "One moment!", said the genie. "Since I am bringing you great riches, you should share some of your gains with me. You must give me 8 coins each time you go around the tree."

Thinking that was a trifling amount, Karim readily agreed.

He went around the tree once. Just as the genie had said, the number of coins in his pocket doubled! He gave 8 coins to the genie. He made another round. Again the number of coins doubled. He gave 8 more coins to the genie. He went around the tree for the third time. The number of coins doubled again, but to his horror, he was left with only 8 coins, exactly the number of coins he owed the genie!

As Karim began to wonder how the genie tricked him, the genie let out a loud laugh and disappeared.

(i) How many coins did Karim initially have?

(ii) For what cost per round should Karim agree to the deal, if he wants to increase the number of coins he has?

(iii) Through its magical powers, the genie knows the number of coins that Karim has. How should the genie set the cost per round so that it gets all of Karim's coins?

Math Talk

SUMMARY

- Algebra is very useful in modeling and understanding numerical scenarios. Because of this, it occurs in almost all areas of mathematics, science and beyond.
- Algebra is an indispensable tool in justifying mathematical statements.
- We applied algebra to analyse 'Think of a Number' tricks, number pyramids, grids, ways of forming numbers using given digits to maximise certain products, divisibility tricks, and various other problems.